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Effect of strain rate on the mechanical behaviors of SiC fiber

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In the present paper, tensile experiments of SiC fiber bundles under different strain rates (quasi-static: 10^{-4}–10^{-3} s^{-1}, dynamic: 200–1200 s^{-1}) are carried out and the corresponding stress-strain curves are obtained. It is found that the mechanical properties of SiC fiber bundles are rate-dependent: the elastic modulus E, strength σ_b and the failure strain ε_b remain unchanged under quasi-static condition, while they apparently increase with increasing strain rate under dynamic condition. Based on the fiber bundles model and the statistical theory of fiber strength, a bi-modal Weibull statistical model of the strain rate dependence is adopted to describe the strength distribution of SiC fiber, and the Weibull parameters are obtained by the fiber bundles testing method. Consistency between the simulated and experimental results indicates that the model and the method are valid and reliable.

1. Introduction

Continuous SiC fiber has been an attractive reinforcement for the ceramic and metal matrix composites due to its advanced mechanical properties (high specific strength, good elevated temperature properties and so on) and the good compatibility with the matrixes. So far, a lot of studies have been focused on the quasi-static mechanical properties of SiC fiber [1, 2] and little on its dynamic behaviors. Since it is inevitable for the fiber-reinforced composites to confront with the dynamic loading and the fiber is the main load-bearing element, it is important to have an understanding of the fiber strength at high strain rate.

Usually the strength of the single fiber follows a statistical distribution and the quasi-static statistical parameters can be obtained by either of two experimental procedures, i.e., single fiber testing and fiber bundle testing [3]. However, for the dynamic parameters, owing to the technical difficulties, there was little until Xia *et al.* [4, 5] successfully performed tensile impact tests on fiber bundles, and extended the method presented by Chi *et al.* [3] for determining quasi-static mechanical parameters of single fiber through fiber bundles testing to the dynamic condition. Wang *et al.* [6] established a bi-modal Weibull distribution model for strain rate and temperature-dependent fiber strength. The method for determining mechanical parameters of fiber by tensile impact tests of fiber bundles is therefore established.

Based on the above testing method and models, the effect of the strain rate on the SiC fiber is studied and characterized in the present paper.

2. Tensile experiments

2.1. Experiment procedure

The test material is Nicalon SiC fiber bundles, which is manufactured by Nippon Carbon Co. Ltd., with the trademark NLM202. Each bundle contains 500 fibers and has a cross-sectional area of 8.667×10^{-2} mm^{-2}. The density of the fiber is 2.537 g/cm^3. Quasi-static (with the strain rate ranging from 0.0001 to 0.001 s^{-1}) and dynamic tensile test (with the strain rate ranging from 200 to 1200 s^{-1}) were performed in a Shimazu DT-5000 testing machine and a self-designed Separated Hopkinson Tensile Bar [7], respectively. The fiber bundles specimen and its connection with the bars are shown in Fig. 1. First the lining block (1) are glued on the supplement plate (2) perpendicularly, 18 fiber bundles (3) are wound in parallel onto the lining blocks, so to make a fiber bundles specimen. In the quasi-static test, the specimen is glued to the slots of two short metal bars (4) and (5) using a high shear strength adhesive, the short metal bars are clamped by the collets of Shimazu DT-5000 during testing; in the dynamic test, the specimen is glued to the slots in the ends of the input bar (4) and output bar (5) with the adhesive. The supplement plate is taken off before testing.

2.2. Experimental results and discussion

The stress-strain curves of SiC fiber bundles at different strain rates are shown in Fig. 2, and the mechanical parameters in Fig. 3 and Table I. In the test at $\dot{\varepsilon} = 0.0001$ s^{-1} and $\dot{\varepsilon} = 0.001$ s^{-1}, the integral

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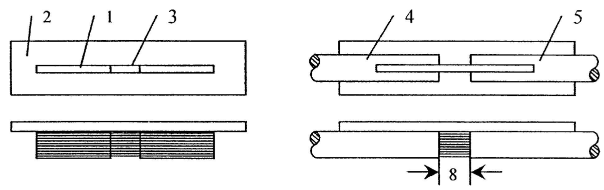

Figure 1 The fiber bundles specimen and its connection: 1. lining block, 2. supplement plate, 3. fiber bundles, 4. input bar (or short metal bar), and 5. output bar (or short metal bar).

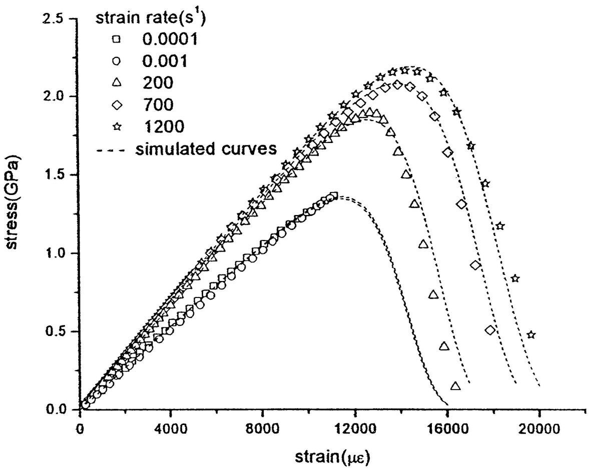

Figure 2 Stress-strain curves of SiC fiber bundles under different strain rates.

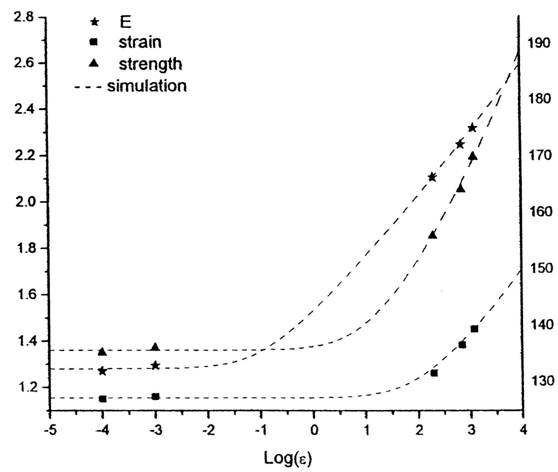

Figure 3 Mechanical properties of SiC fiber versus log(strain rate).

TABLE I Mechanical properties of SiC fiber under different strain rates

$\dot{\varepsilon}$ (s^{-1})	E (GPa)	$\lvert\Delta E/E\rvert$	ε_B (%)	$\lvert\Delta\varepsilon_B/\varepsilon_B\rvert$	σ_b (GPa)	$\lvert\Delta\sigma_b/\sigma_b\rvert$
10^{-4}	132	3.0%	1.15	1.7%	1.35	2.2%
10^{-3}	133	2.3%	1.16	2.6%	1.37	1.5%
200	166	2.3%	1.26	3.2%	1.85	2.7%
700	172	0.9%	1.38	1.4%	2.05	1.5%
1200	175	1.2%	1.45	0.7%	2.19	2.7%

stress-strain curves were not recorded because of the limit on the recording velocity of the Shimazu DT-5000 testing machine. From Fig. 3 it can be seen that the SiC fiber is rate-dependent, the elastic modulus E, strength σ_b and the failure strain ε_b remain unchanged under quasi-static condition, but apparently increase with strain rate under dynamic conditions. Hence, the relationship between these parameters and the strain rate can be simulated as the following exponential function:

$$\sigma_b = \sigma_{b0}\left(\frac{\dot{\varepsilon}+\dot{\varepsilon}_{\sigma t}}{\dot{\varepsilon}_0}\right)^{m_\sigma} \qquad \varepsilon_b = \varepsilon_{b0}\left(\frac{\dot{\varepsilon}+\dot{\varepsilon}_{\varepsilon t}}{\dot{\varepsilon}_0}\right)^{m_\varepsilon}$$
$$E = E_0\left(\frac{\dot{\varepsilon}+\dot{\varepsilon}_{Et}}{\dot{\varepsilon}_0}\right)^{m_E} \tag{1}$$

where $\dot{\varepsilon}$, $\dot{\varepsilon}_0$, σ_{b0}, ε_{b0} and E_0 are strain rate, reference strain rate, reference strength and reference modulus, respectively. m_σ, m_ε and m_E are the rate sensitivity coefficients. $\dot{\varepsilon}_{\sigma t}$, $\dot{\varepsilon}_{\varepsilon t}$ and $\dot{\varepsilon}_{Et}$ are the transition strain rates. The mechanical properties of SiC fiber is highly rate-sensitive when the strain rate exceeds a transition strain rate, while remaining unchanged when the strain rate is below the transition strain rate. Assuming $\dot{\varepsilon} = 200\ s^{-1}$, we obtain the simulated curves by using the least squares method:

$$\sigma_b = 1.85 \times \left(\frac{\dot{\varepsilon}+7.06}{200}\right)^{0.09201}$$
$$\varepsilon_b = 1.26 \times \left(\frac{\dot{\varepsilon}+64.98}{200}\right)^{0.0774}$$
$$E = 166.3 \times \left(\frac{\dot{\varepsilon}+0.086}{200}\right)^{0.02934} \tag{2}$$

Dashed lines in Fig. 3 are simulated results, which fit with experimental data very well.

3. Statistical constitutive model of the fiber bundles

To characterize the strength distribution of SiC fiber, a fiber bundles model is presented in this paper, as shown in Fig. 4. In this model the N parallel fibers of the same length L, cross-sectional area A, are rigidly fixed between the two ends. The following three terms are assumed:

- Each fiber remains completely elastic until it ruptures when the tensile force in the fiber reaches its rupture strength
- The interaction between fibers is neglected. As n single fibers break, the residual load is equally applied to the $N - n$ surviving unbroken fibers. The load and stress of the fiber bundles can be described

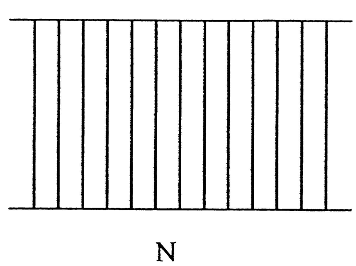

Figure 4 Model of fiber bundles.

as:

$$P = E\varepsilon A(N - n) \tag{3}$$

$$\sigma = E\varepsilon\left(1 - \frac{n}{N}\right) \tag{4}$$

- The strength of the single fiber is satisfactorily given by a particular probabilistic distribution. For failure analysis of a brittle material, the weakest link approach is usually adopted as a criterion of failure; that is, a brittle material fails when the stress at any one flaw becomes larger than the ability of surrounding material to resist local stress. The cumulative distribution function can be given by:

$$G(\varepsilon) = \frac{n}{N} = 1 - \exp\left[-\left(\frac{E\varepsilon}{\sigma_0}\right)^{\beta}\right] \tag{5}$$

where β is the shape parameter and σ_0 is the scale parameter. In recent years, however, it was proposed that the distribution should be given by the modified modal Weibull distribution based on the multi-risk model, if two or more kinds of strength-limiting defect populations exist together in brittle material. For two concurrent flaw populations, a bimodal Weibull distribution function is described as following:

$$G(\varepsilon) = \frac{n}{N} = 1 - \exp\left[-\left(\frac{E\varepsilon}{\sigma_{01}}\right)^{\beta_1} - \left(\frac{E\varepsilon}{\sigma_{02}}\right)^{\beta_2}\right] \tag{6}$$

Then the stress-strain curve of fiber bundles can be rewritten as:

$$\sigma = E\varepsilon\exp\left(-\left(\frac{E\varepsilon}{\sigma_0}\right)^{\beta}\right)$$

(single Weibull distribution) (7)

$$\sigma = E\varepsilon\exp\left[-\left(\frac{E\varepsilon}{\sigma_{01}}\right)^{\beta_1} - \left(\frac{E\varepsilon}{\sigma_{02}}\right)^{\beta_2}\right]$$

(bi-modal Weibull distribution) (8)

Equations 7 and 8 are statistical constitutive equations for single Weibull distribution and bi-modal Weibull distribution, respectively. For single Weibull distribution, we take double logarithms on both sides of Equation 7, i.e.,

$$\mathrm{Ln}\left[-\mathrm{Ln}\left(\frac{\sigma}{E\varepsilon}\right)\right] = \beta\mathrm{Ln}(E\varepsilon) - \beta\mathrm{Ln}\sigma_0 \tag{9}$$

Based on Equation 9, the stress-strain curve of fiber bundles can be rewritten to a straight line in logarithm coordinate system, and β and σ_0 can be determined according to the slope and intercept of the straight line.

By taking double logarithms on both sides of both sides of Equation 8 one can obtain:

$$\mathrm{Ln}\left[-\mathrm{Ln}\left(\frac{\sigma}{E\varepsilon}\right)\right] = \mathrm{Ln}\left[-\left(\frac{E\varepsilon}{\sigma_{01}}\right)^{\beta_1} - \left(\frac{E\varepsilon}{\sigma_{02}}\right)^{\beta_1}\right] \tag{10}$$

The non-linear parameters $\sigma_{01}, \sigma_{02}, \beta_1$ and β_2 can be determined by the regression analysis method [8]. Let

$$y = \mathrm{Ln}[-\mathrm{Ln}(\sigma/E\varepsilon)] \tag{11}$$

$$x = \sigma \tag{12}$$

$$b_1 = \sigma_{01}, b_2 = \sigma_{02}, b_3 = \beta_1, b_4 = \beta_2 \tag{13}$$

substitute Equations 11–13 into Equation 10:

$$y = \mathrm{Ln}\left[\left(\frac{x}{b_1}\right)^{b_3} + \left(\frac{x}{b_1}\right)^{b_4}\right] \tag{14}$$

and

$$\mathrm{d}y = -\frac{b_3/b_1}{1 + u_2/u_1}\mathrm{d}b_1 - \frac{b_4/b_2}{1 + u_1/u_2}\mathrm{d}b_2 + \frac{\mathrm{Ln}(x/b_1)}{1 + u_2/u_1}\mathrm{d}b_3 + \frac{\mathrm{Ln}(x/b_2)}{1 + u_1/u_2}\mathrm{d}b_4 \tag{15}$$

where

$$u_1 = \left(\frac{x}{b_1}\right)^{b_3}, \quad u_2 = \left(\frac{x}{b_2}\right)^{b_4} \tag{16}$$

From experimental data, the value of the modulus E can be estimated. Every bi-modal Weibull Plot curve can be regarded as a combination of two straight lines, each of which provides initial values of b_1, b_2, b_3 and b_4. In Equation 15

$$\mathrm{d}y = y_a - y_b \tag{17}$$

where y_a is the result of Equation 11 and y_b is the result of Equation 14. Each curve contains hundreds of points, so the value of $\mathrm{d}b_i$ ($i = 1, 2, 3, 4$) can be obtained from Equation 15 by using the least-squares method. The values of $b_i + \mathrm{d}b_i$ ($i = 1, 2, 3, 4$) can be seen as initial values and the steps mentioned above can be iterated until $|\mathrm{d}b_i|$ ($i = 1, 2, 3, 4$) are less than certain given small values.

On the basis of Equation 9 and stress-strain curves of fiber bundles, typical Weibull probability plots of SiC fiber under different strain rates are drawn in Fig. 5. It can be seen from Fig. 5 that the Weibull plot of SiC fiber appear approximately bi-linear under high strain rates, which means there are probably two kinds of strength-limiting defect populations exist together in the SiC fiber [7, 9, 10]. Hence, in the present paper, a bi-modal Weibull model is adopted to describe the statistical damage tensile procedure of the fiber bundles.

According to the above Weibull plot, one can obtain the Weibull distribution parameters of the fiber under

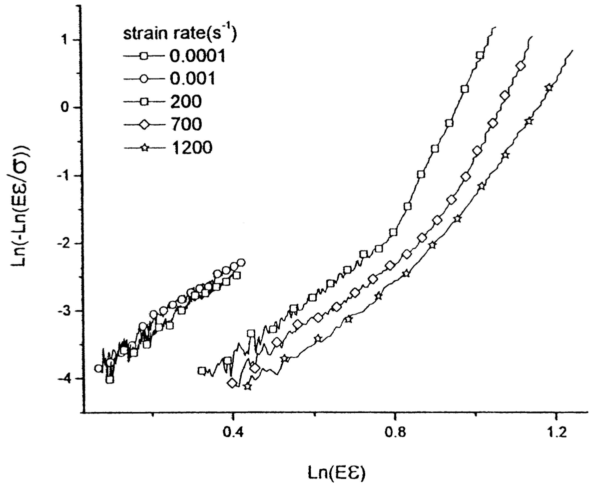

Figure 5 Weibull plot for SiC fiber bundles under different strain rates.

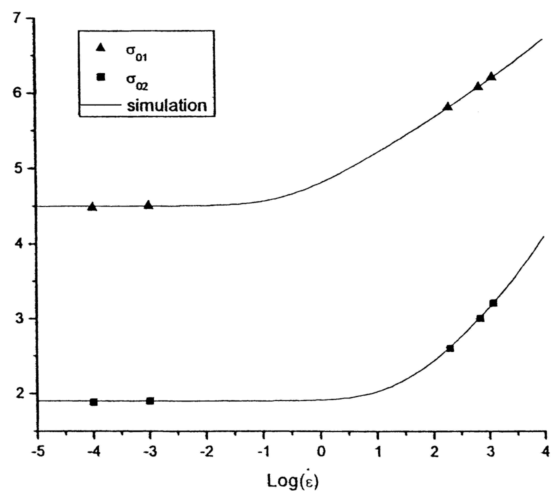

Figure 6 σ_{01} and σ_{02} versus log(strain rate).

dynamic condition. Under the quasi-static loading condition, because the stress-strain curves are not integral, we can only get the low part of the Weibull plots, i.e., the strength distribution of those fiber with relative low strength. In Table II the values of β_1 and β_2 at the quasi-static condition are assigned with the average values of β_1 and β_2 at the dynamic condition, according to the theory that the shape parameters β_1 and β_2 are rate-independent [6]. Then the scale parameters σ_{01} and σ_{02} can be calculated based on the Weibull plot and the given β_1 and β_2.

From Table II it can be seen that the scale parameters σ_{01} and σ_{02} change little under quasi-static condition and apparently increase with strain rate under dynamic condition. Hence we also simulate the relationship of σ_{01}, σ_{02} and strain rate as the following exponential function:

$$\sigma_{01} = 3.75\left(\frac{\dot{\varepsilon} + 26}{200}\right)^{0.0340}$$

$$\sigma_{02} = 2.52\left(\frac{\dot{\varepsilon} + 40}{200}\right)^{0.1140} \quad (17)$$

Substituting the average values of β_1 and β_2, Equation 17 and Equations 2 into 8, the constitutive function of the fiber bundles can be established, the dashed lines in Fig. 2 are the simulated results, which fit the experimental data well.

TABLE II Weibull distribution parameters at different strain rates (specimen gage length = 8 mm)

ε (s^{-1})	σ_{01}	β_1	σ_{02}	β_2
10^{-4}	4.48	2.75	1.88	12.3
10^{-3}	4.50	2.75	1.90	12.3
200	5.80	2.77	2.60	12.5
700	6.07	2.75	3.00	12.2
1200	6.2	2.73	3.20	12.1
Average	/	2.75	/	12.3

4. Conclusion

- SiC fiber is a rate-dependent material. Its elastic modulus E, strength σ_b and the failure strain ε_b of the fiber bundles remain unchanged under quasi-static condition (10^{-4}–10^{-3} s^{-1}), while apparently increase with increasing strain rate under dynamic condition (200–1200 s^{-1}).
- The statistical results show that the dynamic strength distribution of SiC fiber complies with the bi-modal Weibull distribution. The bi-modal Weibull constitutive model can describe the stress-strain relationship of the fiber bundles under different strain rates. The relationship between scale parameters σ_{01}, σ_{02} and the strain rate can be expressed as an exponential function.

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